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Guesswork for Dirac and Majorana neutrino mass matrices*

Wojciech Królikowski

Institute of Theoretical Physics, Warsaw University

Hoża 69, PL–00–681 Warszawa, Poland

Abstract

In the framework of seesaw mechanism with three neutrino flavors, we propose tentatively an efficient parametrization for the spectra of Dirac and righthanded Majorana neutrino mass matrices in terms of three free parameters. Two of them are related to (and determined by) the corresponding parameters introduced previously for the mass spectra of charged leptons and up and down quarks. The third is determined from the experimental estimate of solar Δm^2_{21}. Then, the atmospheric Δm^2_{32} is *predicted* close to its experimental estimation. With the use of these three parameters all light active-neutrino masses $m_1 < m_2 < m_3$ and heavy sterile-neutrino masses $M_1 < M_2 < M_3$ are readily evaluated. The latter turn out much more *hierarchical* than the former. The lightest heavy mass M_1 comes out to be of the order $O(10^6\ \mathrm{GeV})$ so, it is too light to imply that the mechanism of baryogenesis through thermal leptogenesis might work.

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Some time ago we proposed for charged leptons $e_i = e^-, \mu^-, \tau^-$ an efficient empirical mass formula [1]

$$m_{e_i} = \rho_i \mu^{(e)} \left(N_i^2 + \frac{\varepsilon^{(e)} - 1}{N_i^2} \right) , \tag{1}$$

where

$$N_i = 1, 3, 5 , \tag{2}$$

and

$$\rho_i = \frac{1}{29} , \frac{4}{29} , \frac{24}{29} \tag{3}$$

($\sum_i \rho_i = 1$), while $\mu^{(e)} > 0$ and $\varepsilon^{(e)} > 0$ are constants. In fact, with the experimental values $m_e = 0.510999$ MeV and $m_\mu = 105.658$ MeV as an input, the formula (1) rewritten explicitly as

$$m_e = \frac{\mu^{(e)}}{29} \varepsilon^{(e)} \;,\; m_\mu = \frac{\mu^{(e)}}{29} \frac{4}{9} (80 + \varepsilon^{(e)}) \;,\; m_\tau = \frac{\mu^{(e)}}{29} \frac{24}{25} (624 + \varepsilon^{(e)}) \tag{4}$$

leads to the *prediction*

$$m_\tau = \frac{6}{125} (351 m_\mu - 136 m_e) = 1776.80 \text{ MeV} \tag{5}$$

and also determines both constants

$$\mu^{(e)} = \frac{29(9m_\mu - 4m_e)}{320} = 85.9924 \text{ MeV} \;,\; \varepsilon^{(e)} = \frac{320 m_e}{9m_\mu - 4m_e} = 0.172329 \,. \tag{6}$$

The prediction (5) lies really close to the experimental value $m_\tau^{\rm exp} = 1776.99^{+0.29}_{-0.26}$ MeV [2]. Though the formula (1) has essentially the empirical character, there is a speculative background for it based on a Kähler-like extension of Dirac equation which the interested reader may find in Ref. [1]. In particular, the numbers N_i and ρ_i $(i = 1, 2, 3)$ given in Eqs. (2) and (3) are interpreted there.

The charged-lepton mass formula [1] was recently extended to up and down quarks, $u_i = u, c, t$ and $d_i = d, s, b$, by introducing an additional term for the third quark generation, leading to [3]

$$m_{u_i} = \rho_i \mu^{(u)} \left(N_i^2 + \frac{\varepsilon^{(u)} - 1}{N_i^2} + \delta_{i\,3}\beta^{(u)} \right) \tag{7}$$

and

$$m_{d_i} = \rho_i \mu^{(d)} \left(N_i^2 + \frac{\varepsilon^{(d)} - 1}{N_i^2} + \delta_{i\,3}\beta^{(d)} \right) , \tag{8}$$

where N_i and ρ_i are given as before in Eqs. (2) and (3), while $\mu^{(u,d)} > 0$, $\varepsilon^{(u,d)} > 0$ and $\beta^{(u,d)} > 0$ are constants. It is seen that *a priori* Eqs. (7) and (8) cannot give us any mass predictions, since there are six quark masses and six free parameters. However, the latter are uniquely determined. In fact, assuming for quark masses their mean experimental estimates [2]

$$m_u \sim 3 \text{ MeV} \quad , \quad m_c \sim 1.2 \text{ GeV} \quad , \quad m_t \sim 174 \text{ GeV} \tag{9}$$

and

$$m_d \sim 6.75 \text{ MeV} \quad , \quad m_s \sim 118 \text{ MeV} \quad , \quad m_b \sim 4.25 \text{ GeV} \, , \tag{10}$$

one can calculate

$$m_{t,b} = \frac{6}{125}(351 m_{c,s} - 136 m_{u,d}) + \frac{24}{29}\mu^{(u,d)}\beta^{(u,d)} \sim \left\{ \begin{array}{c} 20 \quad + \, 0.81 \; \beta^{(u)} \\ 1.94 + 0.078 \, \beta^{(d)} \end{array} \right\} \text{GeV} \tag{11}$$

and

$$\mu^{(u,d)} = \frac{29(9m_{c,s} - 4m_{u,d})}{320} \sim \left\{ \begin{array}{c} 978 \\ 93.8 \end{array} \right\} \text{MeV} \, , \; \varepsilon^{(u,d)} = \frac{320 m_{u,d}}{9m_{c,s} - 4m_{u,d}} \sim \left\{ \begin{array}{c} 0.0890 \\ 2.09 \end{array} \right\} . \tag{12}$$

From Eqs. (11) it follows that

$$\beta^{(u)} \sim 190 \, , \; \beta^{(d)} \sim 30 \, , \tag{13}$$

thus

$$\frac{\beta^{(u)}}{\beta^{(d)}} \sim 6.3 \, . \tag{14}$$

It may be interesting to note that the experimental ratio (14) is nicely reproduced by the ansatz

$$\beta^{(u,d)} \propto (3B + Q^{(u,d)})^2 = \left\{ \begin{array}{r} 25/9 \\ 4/9 \end{array} \right\} , \tag{15}$$

where $B = 1/3$ and $Q^{(u,d)} = \left\{ \begin{array}{r} 2/3 \\ -1/3 \end{array} \right\}$ are the baryon number and electric charge of quarks. Then, $\beta^{(u)}/\beta^{(d)} = 6.25$. Note also that the analogical constant for charged leptons vanishes, $\beta^{(e)} \propto (L + Q^{(e)})^2 = 0$, where $L = 1$ and $Q^{(e)} = -1$ are their lepton number and electric charge ($F = 3B + L$ is the fermion number as defined for quarks and charged leptons).

In the present note we discuss the mass spectrum of three active (lefthanded) neutrinos $\nu_{\alpha L} = \nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$ related to their mass states $\nu_{iL} = \nu_{1L}, \nu_{2L}, \nu_{3L}$ through the unitary mixing transformation

$$\nu_{\alpha L} = \sum_i U_{\alpha i}\, \nu_{iL} \;, \tag{16}$$

where the neutrino mixing matrix $U = (U_{\alpha i})$ is experimentally consistent with the bilarge form [4]

$$U = \left(\begin{array}{ccc} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}} s_{12} & \frac{1}{\sqrt{2}} c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} s_{12} & -\frac{1}{\sqrt{2}} c_{12} & \frac{1}{\sqrt{2}} \end{array} \right) , \tag{17}$$

where $c_{12} = \cos\theta_{12}$ and $s_{12} = \sin\theta_{12}$ with large $\theta_{12} \sim 33^\circ$, while $c_{23} = \cos\theta_{23} = 1/\sqrt{2}$ and $s_{23} = \sin\theta_{23} = 1/\sqrt{2}$ with maximal $\theta_{23} = 45^\circ$. In Eq. (17) the matrix element $U_{e3} = s_{13}\exp(-i\delta)$ is neglected, where $s_{13} = \sin\theta_{13}$ with $s^2_{13} < 0.03$. Three sterile (righthanded) neutrinos $\nu_{\alpha R} = \nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ and their mass states $\nu_{iR} = \nu_{1R}, \nu_{2R}, \nu_{3R}$ appear as a background.

Our starting point will be the generic 6×6 mass matrix

$$\left(\begin{array}{cc} 0 & M^{(D)} \\ M^{(D)\,T} & M^{(R)} \end{array} \right) \tag{18}$$

(in the basis of active $\nu_{\alpha L}$ and sterile $\nu_{\alpha R}$), involving Dirac and righthanded Majorana 3×3 mass matrices, $M^{(D)}$ and $M^{(R)}$. Accepting the familiar seesaw mechanism [5] we will use for active neutrinos the effective Majorana 3×3 mass matrix of the form

$$M^{(\nu)} = M^{(D)} M^{(R)^{-1}} M^{(D)\,T}\,, \tag{19}$$

where $M^{(R)}$ is assumed to dominate over $M^{(D)}$. For the eigenvalues $m^{(D)}_{\nu_i} = m^{(D)}_{\nu_1}$, $m^{(D)}_{\nu_2}, m^{(D)}_{\nu_3}$ of the Dirac neutrino mass matrix $M^{(D)}$ we will accept tentatively the formula of the same type as Eq. (1) for charged leptons,

$$m^{(D)}_{\nu_i} = \rho_i \mu^{(\nu)} \left(N_i^2 + \frac{\varepsilon^{(\nu)} - 1}{N_i^2} \right)\,, \tag{20}$$

where $\mu^{(\nu)} > 0$ and $\varepsilon^{(\nu)} > 0$ are new constants.

In the flavor representation, where the charged-lepton mass matrix $M^{(e)}$ is diagonal, the neutrino mixing matrix U is at the same time the neutrino diagonalizing matrix,

$$U^\dagger M^{(\nu)} U = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\,. \tag{21}$$

Here, for simplicity, $M^{(\nu)*} = M^{(\nu)}$ and $U^* = U$ [as in Eq. (17)]. In the case of seesaw form (19) of $M^{(\nu)}$, it seems natural to conjecture that U is also the diagonalizing matrix for the Dirac neutrino mass matrix $M^{(D)}$,

$$U^\dagger M^{(D)} U = \mathrm{diag}(m^{(D)}_{\nu_1}, m^{(D)}_{\nu_2}, m^{(D)}_{\nu_3})\,. \tag{22}$$

Notice that then the inverse $M^{(R)^{-1}}$ in Eq. (19) is diagonalized by U as well, since from Eq. (19) $M^{(R)^{-1}} = M^{(D)^{-1}} M^{(\nu)} M^{(D)T^{-1}}$ (if the inverse $M^{(D)^{-1}}$ exists *i.e.*, all $m^{(D)}_{\nu_i} \neq 0$) and both $M^{(\nu)}$ and $M^{(D)^{-1}}$ on the rhs are diagonalized by U. Hence

$$U^\dagger M^{(R)} U = \mathrm{diag}(M_{\nu_1}, M_{\nu_2}, M_{\nu_3})\,, \tag{23}$$

when Eq. (22) is conjectured.

Thus, under the conjecture (22) we obtain from the seesaw form (19) of $M^{(\nu)}$ the following Majorana mass spectrum for light active (lefthanded) neutrinos ν_{iL}:

$$m_{\nu_i} = \frac{m^{(D)2}_{\nu_i}}{M_{\nu_i}} \tag{24}$$

with $M_{\nu_i} \gg m^{(D)}_{\nu_i} \gg m_{\nu_i}$, where M_{ν_i} are the Majorana masses of heavy sterile (righthanded) neutrinos ν_{iR}. Here, for simplicity, $M^{(R)*} = M^{(R)}$.

In order to proceed further with calculations of $m_{\nu_i}^{(D)}$ [from Eq. (20)] and m_{ν_i} [from Eq. (24)] we are forced to make some tentative conjectures about $\mu^{(\nu)}$ and $\varepsilon^{(\nu)}$ as well as M_{ν_i}. We will propose tentatively that

$$\mu^{(\nu)} \ : \mu^{(e)} = \mu^{(u)} : \mu^{(d)} \ , \ \varepsilon^{(\nu)} : \varepsilon^{(e)} = \varepsilon^{(u)} : \varepsilon^{(d)} \tag{25}$$

and also

$$M_{\nu_i} \propto N_i^2 m_{\nu_i}^{(D)} \,, \tag{26}$$

where $N_i = 1, 3, 5$ as before in Eq. (2) [in Ref. [6] we conjectured tentatively that $M_{\nu_i} \propto N_i^2 m_{e_i}$ instead of Eqs (26)]. With the use of values (6) and (12), Eqs. (25) imply that

$$\mu^{(\nu)} \sim 896 \text{ MeV} \ , \ \varepsilon^{(\nu)} \sim 7.35 \times 10^{-3} \,. \tag{27}$$

Then, the mass formula (20) gives the following *hierarchical* estimates of Dirac neutrino masses:

$$\begin{aligned}
m_{\nu_1}^{(D)} &= \frac{\mu^{(\nu)}}{29} \varepsilon^{(\nu)} \sim 0.227 \text{ MeV} \ll \frac{\mu^{(\nu)}}{\mu^{(e)}} m_e \,, \\
m_{\nu_2}^{(D)} &= \frac{\mu^{(\nu)}}{29} \frac{4}{9} \left(80 + \varepsilon^{(\nu)}\right) \sim 1.10 \text{ GeV} \sim \frac{\mu^{(\nu)}}{\mu^{(e)}} m_\mu \,, \\
m_{\nu_3}^{(D)} &= \frac{\mu^{(\nu)}}{29} \frac{24}{25} \left(624 + \varepsilon^{(\nu)}\right) \sim 18.5 \text{ GeV} \sim \frac{\mu^{(\nu)}}{\mu^{(e)}} m_\tau \,.
\end{aligned} \tag{28}$$

The (weighted) proportionality relation (26), when applied to the seesaw spectrum (24), leads to

$$m_{\nu_i} \propto \frac{1}{N_i^2} m_{\nu_i}^{(D)} \,. \tag{29}$$

This shows that m_{ν_i} are less *hierarchical* than $m_{\nu_i}^{(D)}$. From Eqs. (29) we can see that

$$\frac{m_{\nu_1}}{m_{\nu_2}} = 9 \frac{m_{\nu_1}^{(D)}}{m_{\nu_2}^{(D)}} \sim 1.86 \times 10^{-3} \ , \ \frac{m_{\nu_2}}{m_{\nu_3}} = \frac{25}{9} \frac{m_{\nu_2}^{(D)}}{m_{\nu_3}^{(D)}} \sim 0.165 \,. \tag{30}$$

Thus, taking the experimental estimate $m_{\nu_2}^{\rm exp} = \sqrt{(\Delta m_{21}^2)^{\rm exp}} \sim \sqrt{7 \times 10^{-5}}\,\text{eV} = 8.4 \times 10^{-3}\text{eV}$ as an input, we *predict* from Eqs. (30) that

$$m_{\nu_1} \sim 1.6 \times 10^{-5}\ \text{eV}\ ,\ m_{\nu_3} \sim \sqrt{2.6 \times 10^{-3}}\ \text{eV} = 5.1 \times 10^{-2}\ \text{eV}\ . \tag{31}$$

The prediction $m_{\nu_3} \sim \sqrt{2.6 \times 10^{-3}}$ eV gives $\Delta m_{32}^2 = m_{\nu_3}^2 - (m_{\nu_2}^{\text{exp}})^2 \sim 2.5 \times 10^{-3}\ \text{eV}^2$ close to the experimental estimate $(\Delta m_{32}^2)^{\text{exp}} \sim 2.5 \times 10^{-3}\ \text{eV}^2$ (the lower estimate $(\Delta m_{32}^2)^{\text{exp}} \sim 2 \times 10^{-3}\ \text{eV}^2$ would correspond to the lower prediction $m_{\nu_3} \sim \sqrt{2.1 \times 10^{-3}}$ eV).

Denoting the proportionality coefficient in Eq. (26) by ζ, we have

$$M_{\nu_i} = \zeta N_i^2 m_{\nu_i}^{(D)} \tag{32}$$

and $m_{\nu_i} = (1/\zeta) m_{\nu_i}^{(D)}/N_i^2 = (1/\zeta^2) M_{\nu_i}/N_i^4$ due to Eq. (29). Thus, ζ may be calculated *e.g.* from the relation

$$\zeta = m_{\nu_2}^{(D)}/9 m_{\nu_2} \sim 1.46 \times 10^{10}\,, \tag{33}$$

where the experimental estimate $m_{\nu_2}^{\text{exp}} \sim \sqrt{7 \times 10^{-5}}$ eV is applied. Then, using the values (28), we obtain from Eqs. (32) the following *hierarchical* estimates of Majorana sterile-neutrino masses:

$$\begin{aligned} M_{\nu_1} &= \zeta m_{\nu_1}^{(D)} \sim 3.3 \times 10^{6}\ \text{GeV}\,, \\ M_{\nu_2} &= 9\zeta m_{\nu_2}^{(D)} \sim 1.4 \times 10^{11}\ \text{GeV}\,, \\ M_{\nu_3} &= 25\zeta m_{\nu_3}^{(D)} \sim 6.8 \times 10^{12}\ \text{GeV}\,. \end{aligned} \tag{34}$$

It is seen that $m_{\nu_i}^{(D)}$ are less *hierarchical* than M_{ν_i} (and m_{ν_i} less than $m_{\nu_i}^{(D)}$).

Note that the Majorana mass M_{ν_1} of the lightest heavy sterile neutrino ν_{1R} is too light by two orders of magnitude to reach the estimated lower bound $M_{\nu_1} \gtrsim 10^8$ GeV required for the working of baryogenesis through thermal leptogenesis [7] (of course, in this mechanism $M^{(\nu)*} \neq M^{(\nu)}$ and $M^{(R)*} \neq M^{(R)}$).

In conclusion, our tentative proposal presented here for the Dirac and Majorana neutrino mass matrices $M^{(D)}$ and $M^{(R)}$ contains two items: (*i*) the parametrization (20) of Dirac neutrino masses $m_{\nu_i}^{(D)}$ in terms of two constants $\mu^{(\nu)}$ and $\varepsilon^{(\nu)}$ determined through the conditions (25), and (*ii*) the parametrization (32) of Majorana neutrino masses M_{ν_i} by one constant ζ determined from the experimental estimation of solar

Δm^2_{21}. Then, the atmospheric Δm^2_{32} is *predicted* close to its experimental estimate. All neutrino seesaw masses m_{ν_i} and Majorana masses are evaluated. The mass spectra $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$, $m^{(D)}_{\nu_1} < m^{(D)}_{\nu_2} < m^{(D)}_{\nu_3}$ and $M_{\nu_1} < M_{\nu_2} < M_{\nu_3}$ are *hierarchical*, behaving as $1 : 5.4\times10^2 : 3.3\times10^3$, $1 : 4.8\times10^3 : 8.2\times10^4$ and $1 : 4.4\times10^4 : 2.0\times10^6$, respectively, with $m_{\nu_1} \sim 1.6\times10^{-5}$ eV, $m^{(D)}_{\nu_1} \sim 0.23$ MeV, and $M_{\nu_1} \sim 3.3\times10^6$ GeV.

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